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Non-NewtonianFluid Flow and Heat Transfer over a Non-Linearly Stretching Surface Along With Porous Plate in Porous Medium

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ABSTRACT
This paper investigates the MHD flow and heat transfer of an electrically conducting non-newtonian power-law fluid over a non-linearly stretching surface along with porous plate in porous medium. The governing equations are reduced to non-linear ordinary differential equations by means of similarity transformations. These equations are then solved numerically with the help ofRunge – Kutta shooting method. The effect of various flow parameters in the form of dimensionless quantities on the flow field are discussed and presented graphically.
Keywords : Heat Source /Sink Parameter;MHD flow;Power-law Fluid; Stretching sheet; porous medium.

I. Introduction :

The study of non-newtonian fluid flows hasthe gained considerable interest for their numerous engineering applications. Details of the behavior of non-newtonian fluid for both steady and unsteady flow situations along with mathematical models can be found in the books by Astarita and Marrucci[1].Over recent years, applications of non-newtonian fluids in many industrial processes have been increasing. Many particulate slurries, multiphase mixtures, pharmaceutical formulations,cosmetics and toiletries,paints,biological fluids and food items are examples of non- Newtonian fluids.The study of hydrodynamics flow and heat transfer over a stretching sheet may find its applications in polymer technology related to the stretching of plastic sheets.T.Sarpakaya[2]studied flow on non-newtonian fluids in a magnetic field. K.B.Pavolov[3]examined the Magneto-Hydrodynamic flow of an Incompressible viscous fluid caused by deformation of a plan surface.H.I.Andersson,K.H.Bech and B.S.Dandapat[4]described Magneto-Hydrodynamic flow of a power-law fluid over a stretching sheet.R.Cortell[5]described a note on MHD flow of a power-law fluid over a stretching sheet.A.A.Mutlag,Md.Jasminuddin,A.I.Md.Ismail and M.A.A.Hamad[6]described Scaling group Transformation under the effect of Thermal radiation Heat Transfer of a non-newtonian power-law fluid over a vertical stretching sheet with momentum slip boundary condition. The problem of dissipation effects on MHD non-linear flow and heat transfer past a porous surface with prescribed heat flux have been studied by S.P.Anjalidevi and B.Ganga[7]. The effect of viscous and joules dissipation on MHD flow , heat and mass transfer past stretching porous surface embedded in porous medium was studied by S.P.Anjalidevi and B.Ganga[8]. P.K.Singh and Jai singh[9] examined the MHD flow with viscous dissipation and chemical reaction over a stretching porous plate in porous medium. R.Cortell[10] described viscous flow and heat transfer over a non-linearly stretching sheet. More recently,KerehalliVinayaka Prasad, SeetharamanRajeswarisanthi and PampannaSomannaDatti[11]examined the effect of variable thermal conductivity on the power-law fluid flow and heat transfer over a non-linearly stretching sheet in the presence of a transverse magnetic field.

Motivated by these analyses and practical applications, the main concern of the present paper is to study the effect of variable thermal conductivity on the power-law fluid flow and heat transfer over a non-linearly stretching sheet along with porous plate in porous medium in the presence of a transverse magnetic field. This extends the work in [11]. The obtain similarity equations were solved numerically to show the effects of the governing parameters.

II. Nomenclature

A - constant
B_0 - uniform magnetic field
b - stretching rate, positive constant
C_f - skin friction
f - dimensionless stream function
$h(x)$ - heat transfer co-efficient
k - thermal conductivity
K - consistency co-efficient

m - velocity exponent parameter
Mn - magnetic parameter
n - power-law index
Nu_x -Nusselt number
Npr - modified Prandtl number
Npe_x - Peclet number
q_w - local heat flux at the sheet
p -pressure
Re_x -local Reynolds number
r -temperature exponent parameter
T -fluid temperature
$T_w(x)$-temperature of the stretching sheet
T_∞ - ambient temperature
u -velocity in x direction
v - velocity in y direction
$U(x)$ - velocity of the stretching sheet
Q_s -temperature-dependent volumetric rate of heat source
c_p - specific heat at constant pressure
α -thermal diffusivity
β -heat source/sink parameter
$c\nu$ -kinematic viscosity
δ -permeability parameter
η - similarity variable
μ_0 -magnetic permeability
ψ -Stream function
ρ - density
σ - electrical conductivity
θ - dimensionless temperature

III. Mathematical Formulation

Let us consider the case of a steady two-dimensional flow and heat transfer of a non-newtonian power –law fluid over a non -linear stretching sheet. We consider the effect of porous plate in porous medium. Further a Cartesian co-ordinate system(x , y) is used where x and y are co-ordinates measured along and normal to the surface, respectively. The continuous stretching sheet is assumed to have a non-linear velocity and prescribed temperature of the form $U(x) = bx^m$ and $T_\infty + Ax^r$ respectively, where b is the stretching constant, x is the distance from the slot; A is a constant whose value depends upon the properties of the fluid. Here m and r are the velocity and temperature exponents,respectively. Here we neglect the induced magnetic field, which is small in comparison with the applied magnetic field. Further the external electrical field is assumed to be zero. Under these assumptions the basic equations governing the flow and heat transfer in usual notations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (3.1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = -\nu\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^{n-1} - \frac{\sigma B_0^2}{\rho}u - \frac{\nu}{K}u \qquad (3.2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{Q_S}{\rho c_P}\left(T - T_\infty\right) \qquad (3.3)$$

Where u and v are the flow velocity components along the x and y -axes respectively, ν is the kinematic viscosity of the fluid, κ -consistency of the fluid, n is the power-law index, ρ is the fluid density and c_P is the specific heat at constants pressure.

The first term in the right hand side of the equation (3.2) is the shear rate $\left(\partial u/\partial x\right)$ has been assumed to be negative throughout the boundary layer since the stream wise velocity component u decreases monotonically with the distance y from the moving surface(for continuous stretching surface). The flow is driven solely by the stretching surface, which moves with a prescribed velocity $U(x)$. T is the temperature of the fluid and α is the thermal diffusivity of the fluid. The last term containing Q_S in equation (3.3) represents the temperature dependent volumetric rate of heat source when $Q_S > 0$ and heat sink when $Q_S < 0$. Thus the appropriate boundary conditions are

$$u(x,0) = U(x), \qquad (3.4)$$

$$v(x,0) = 0, \qquad (3.5)$$

$$T(x,0) = T_w(x), \qquad (3.6)$$

$$u(x,y) \to 0, T(x,y) \to T_\infty \text{ as } y \to \infty \qquad (3.7)$$

Here, boundary condition (3.4) assures no slip at the surface and equation (3.5) signifies the importance of impermeability of the stretching surface. Equation (3.6) is the variable prescribed surface temperature at the wall whereas the equation (3.7) means that the stream velocity and the temperature vanish outside the boundary layer. In order to obtain the similarity solutions of equations (3.1) - (3.7), we assume that the variable magnetic field $B_0(x)$ is of the form

$B_0(x) = B_0 x^{(m-1)/2}$. The momentum and energy equations can be transformed to the corresponding ordinary differential equations by the following transformations ([11])

$$\eta = \frac{y}{x}\left(\mathrm{Re}_x\right)^{\frac{1}{n+1}}, \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (3.8)$$

$$\psi(x, y) = Ux\left(\mathrm{Re}_x\right)^{\frac{-1}{n+1}} f(\eta), \quad (3.9)$$

Where η is the similarity variable, $\psi(x, y)$ is the stream function f and θ are the dimensionless similarity function and temperature, respectively. The velocity components u and v given by

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \quad (3.10)$$

The local Reynolds number is defined by

$$\mathrm{Re}_x = \frac{U^{2-n} x^n}{\nu} \quad (3.11)$$

The mass conservation equation (3.1) is automatically satisfied by equation (3.10). By assuming the similarity function $f(\eta)$ to depend on the similarity variable η, the momentum equation(3.2) and the heat equation (3.3) transform into the coupled non-linear ordinary differential equation of the form

$$n\left(-f''\right)^{n-1} f''' - mf'^2 + \left(\frac{2mn - m + 1}{n+1}\right) ff'' - \left(Mn + \delta^{-1}\right) f' = 0 \quad (3.12)$$

$$\theta'' + Npr\left(\frac{2mn - m + 1}{n+1}\right) f\theta' + Npr\left(\beta\theta - rf'\theta\right) = 0 \quad (3.13)$$

The boundary conditions(3.4) – (3.7) now becomes

$$f(\eta) = 0, f'(\eta) = 1, \theta(\eta) = 1, at \eta = 0 \quad (3.14)$$

$$f'(\eta) \to 0, \theta(\eta) \to 0 \text{ as } \eta \to \infty \quad (3.15)$$

Where $Mn = \sigma B_0^2 / \rho b$ is the magnetic parameter, $\delta = Kb/\nu$ is the permeability parameter, $Npr = Npe_x / \left(\mathrm{Re}_x\right)^{\frac{2}{n+1}}$ is the modified Prandtl number for power – law fluids, $Npe_x = c_p \rho U x / \kappa$ is the convectional Peclet number[11], $\beta = Q_s / c_p \rho b$ is the heat source/sink parameter. Here primes denote the differentiation with respective to η.

The physical quantities of interest are the skin-friction coefficient C_f and the local Nusselt number Nu_x, which are defined as

$$C_f = \frac{2\tau_w}{\rho U^2}, \quad Nu_x = \frac{xq_w}{k\left(T_w - T_\infty\right)} \quad (3.16)$$

respectively, where the wall shear stress τ_w and heat transfer from the sheet q_w are given by

$$\tau_w = \mu_0 \left(\frac{\partial u}{\partial y}\right)_{at y=0}, q_w = -\kappa\left(\frac{\partial T}{\partial y}\right)_{at y=0} \quad (3.17)$$

with μ_0 and κ being the dynamic viscosity and thermal conductivity, respectively. Using the non-dimensional variable (2.7), we obtain

$$C_f = \left(\frac{-2\tau_{xy}}{\rho (bx)^2}\right)_{y=0} = 2\left[-f''(0)\right]^n \left[\mathrm{Re}_x\right]^{\frac{-1}{n+1}} \quad (3.18)$$

$Nu_x = -\left[\mathrm{Re}_x\right]^{\frac{-1}{n+1}} \theta'(0)$, where τ_{xy} is the shear stress and Re_x is the local Reynolds number.

IV. Results and Discussion

The ordinary differential equations(3.12)-(3.13) with the boundary conditions(3.14)-(3.15)have been solved by Runge-kutta shooting method. Numerical results are obtained to study the effect of the various non-dimensional parameters namely, the magnetic parameters Mn permeability parameter δ, the velocity exponent parameter m and the temperature exponent parameter r ,the modified Prandtl number Npr and the heat source/sink parameter β on the flow and heat transfer are shown graphically in the Figures 1-9.

Figures (1(a)-(c)) respectively, depict the effect of shear thinning $(n < 1)$, newtonian $(n = 1)$ and shear thickening $(n > 1)$ fluids on the horizontal velocity profiles f' with η,for different values of velocity exponent parameter m and magnetic parameter Mn and constant permeability parameter δ. The effect of flattering of horizontal velocity as a consequence of increasing the strength of the magnetic field is observed for all values of velocity exponent parameter m. The effect of increasing values of the velocity exponent parameter m is to reduce the momentum boundary layer thickness, which tends to zero as the space variable η increases from the boundary surface. Physically, $m<0$ implies that the surface is decelerated from the slot, $m=0$ the continuous movement of a flat surface, and $m>0$ implies the surface is accelerated from the extruded slit.

Horizontal velocity profiles f' decreases and disclosing the fact that the effect of stretching of the exponent parameter m from negative values to positive values is to decelerate the velocity and hence reduces the momentum boundary layer thickness. Further it is observed from these,Figures ((1(a)-1(c)) that the horizontal velocity profiles f' decrease with increasing values of power-law index n.

Figures 2((a)-(c)) respectively demonstrates the horizontal velocity profiles for different values of permeability parameter (δ) in the presence/absence of magnetic parameter Mn. It is observed that the velocity of the fluid increases as permeability parameter increases.

Figures 3((a)-(c)) respectively demonstrates the temperature profiles for different values of permeability parameter (δ) in the presence/absence of magnetic parameter Mn. It is observed that the temperature of the fluid decreases as permeability parameter increases.

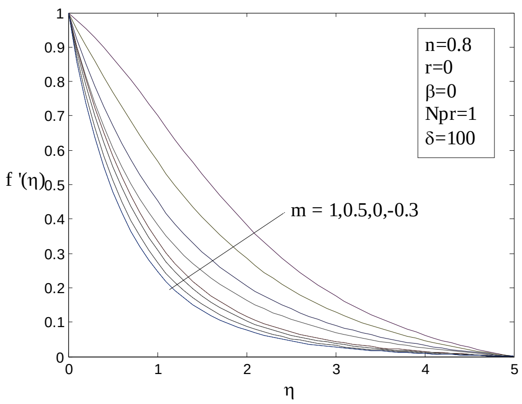
Figures 4((a)-(c)) respectively depicts temperature profiles for different values of velocity exponent parameter (m) in the presence/absence of magnetic parameter Mn. The effect of increasing values of velocity exponent parameter m is to decrease the temperature profiles.

Figures 5((a)-(c)) shows respectively, the shear thinning, Newtonian and shear thickening fluids on the temperature profiles $\theta(\eta)$ for different values of temperature exponent parameter r and velocity exponent parameter m in presence of magnetic parameter Mn. It is seen that the temperature of the fluid decrease as temperature exponent parameter r increases. The same result is obtained for absence of magnetic field also.

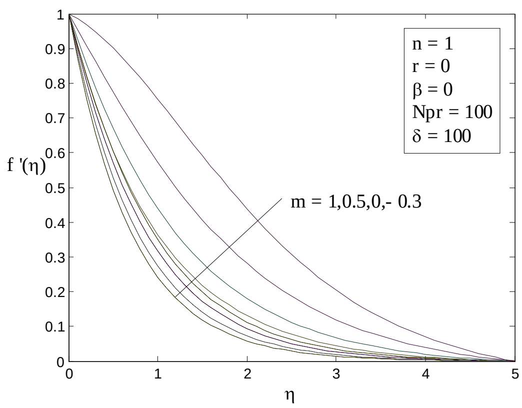
Figures 6((a)-(c)) demonstrates the temperature profiles for several sets of values of the modified Prandtle number Npr in the absence/presence of magnetic parameter Mn for shear thinning, Newtonian and shear thickening fluids. It is clear that the temperature of the fluid decreases asPrandtle numberincreases.

Fig7((a)-(c)) shows respectively , the shear thinning, Newtonian and shear thickening fluids on the temperature profiles for different values of heat source/sink parameter β in the presence of magnetic field Mn. It is clear that temperature profiles increases as heat source/sink parameter β increases.The same result is obtained for absence of magnetic field also.

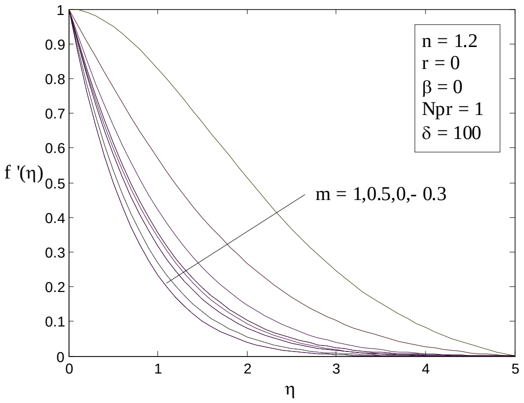
$Mn=0$, $Mn=1$



1(a)

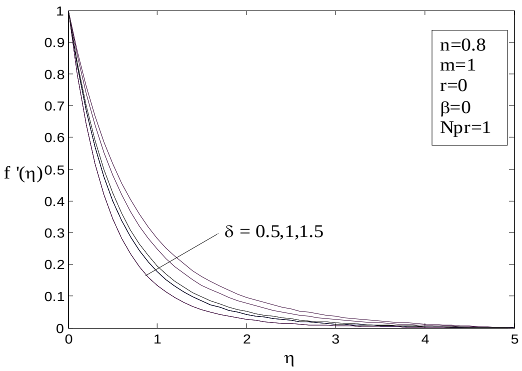


1(b)

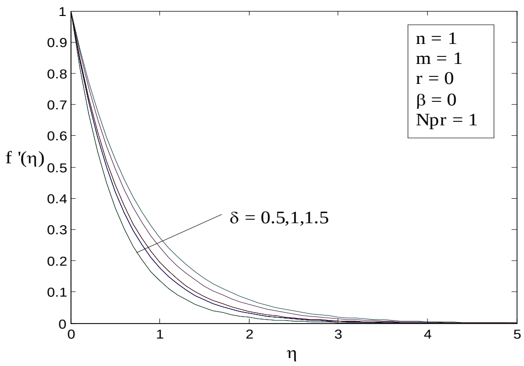


1(c)

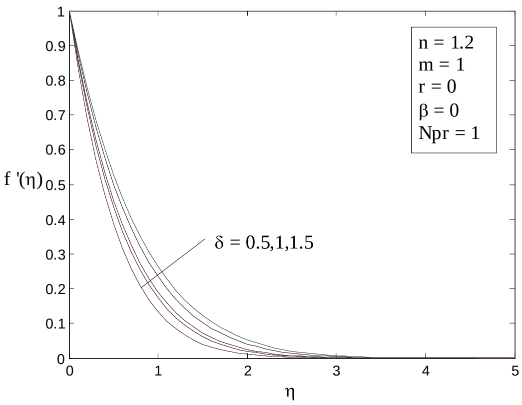
Fig.1.Non dimensional velocity profiles for different values of stretching parameter m



2(a)

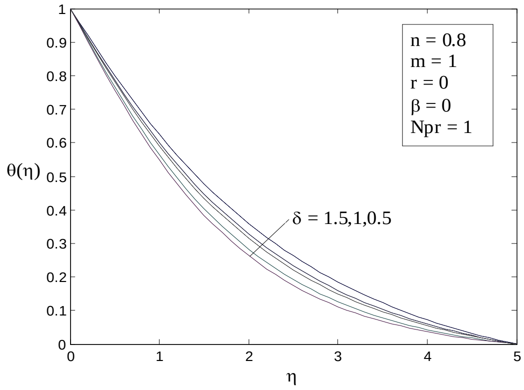


2(b)

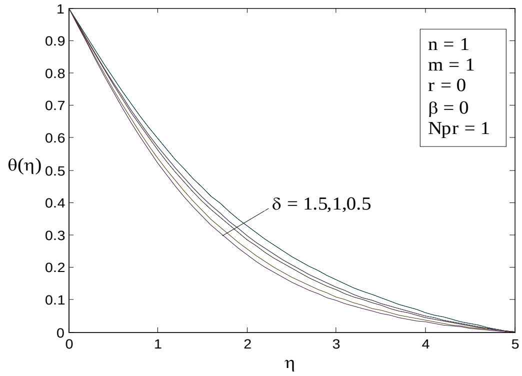


2(c)

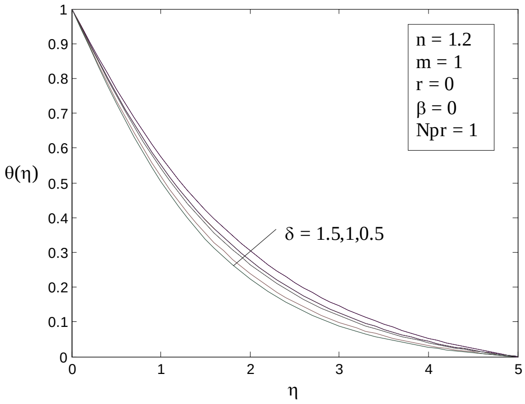
Fig.2.Non dimensional velocity profiles for different values of δ with presence/absence of magnetic field Mn



3(a)



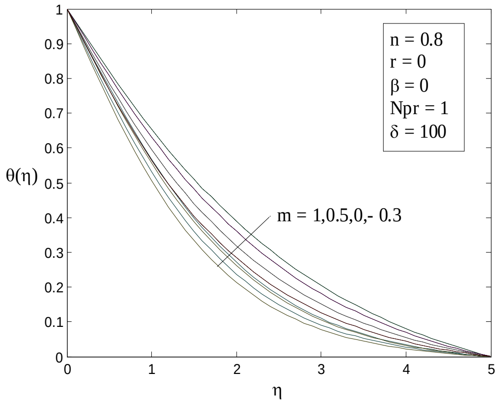
3(b)



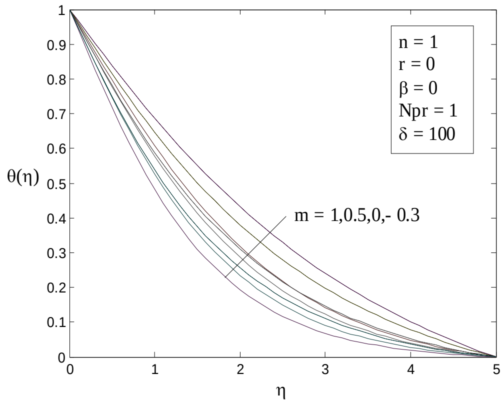
3(c)

Fig.3.Effect of temperature distribution for different values of δ with presence/absence of magnetic parameter

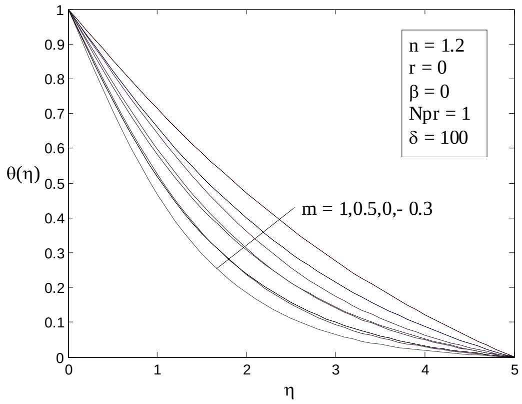
$Mn = 0$, $Mn = 1$



4(a)



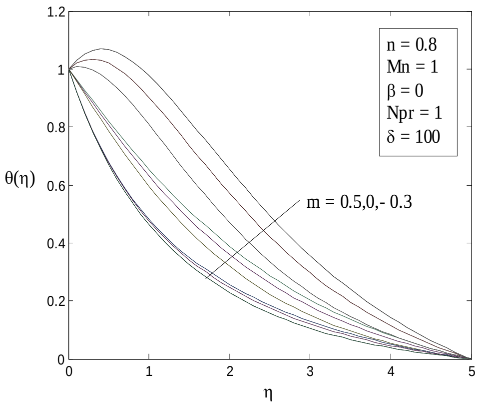
4(b)



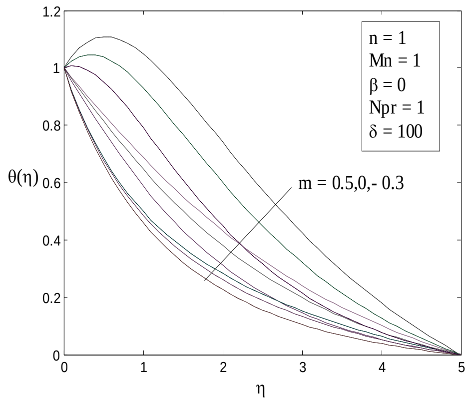
4(c)

Fig.4.Effect of stretching parameter m over temperature distribution

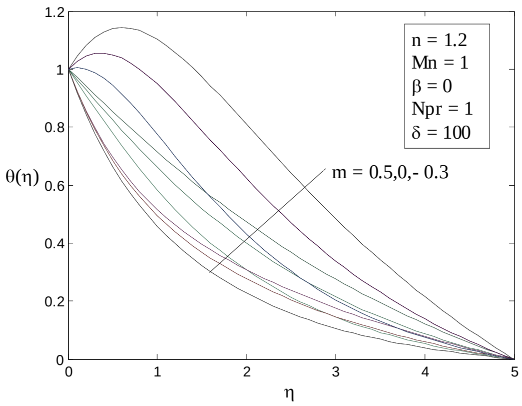
—— $r = 0$, —— $r = 1$, —— - $r = -1$



5(a)



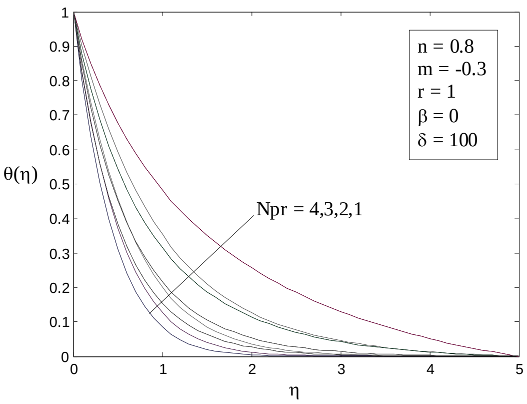
5(b)



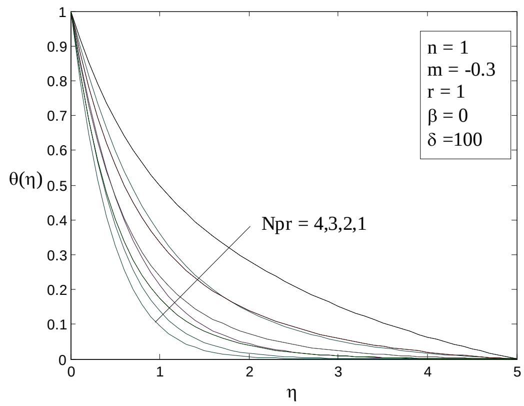
5(c)

Fig.5.Temperature profiles for different values of stretching parameter m and wall temperature r with $Mn = 1$

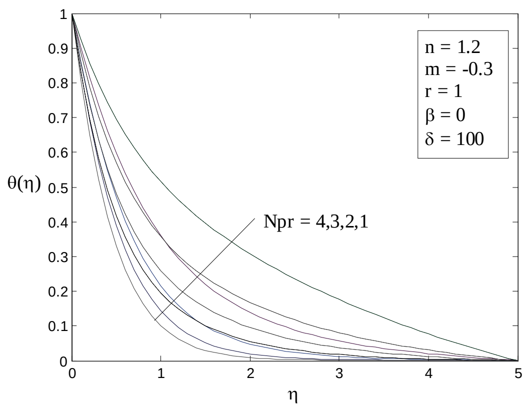
$Mn = 0$, $Mn = 1$



6(a)



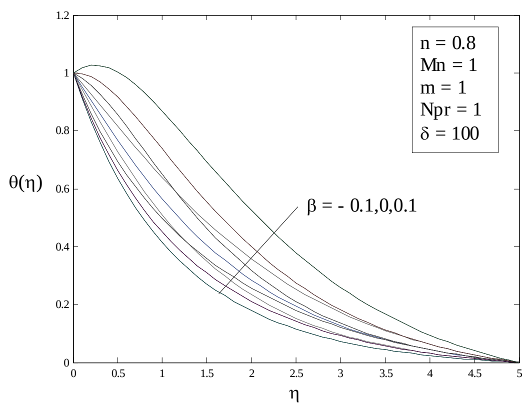
6(b)



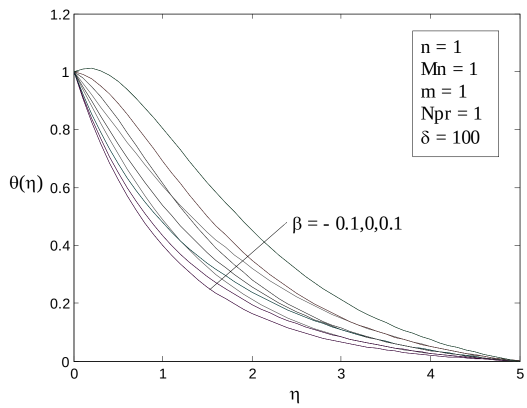
6(c)

Fig.6.Temperature profiles for different values of modified Prandtle number Npr

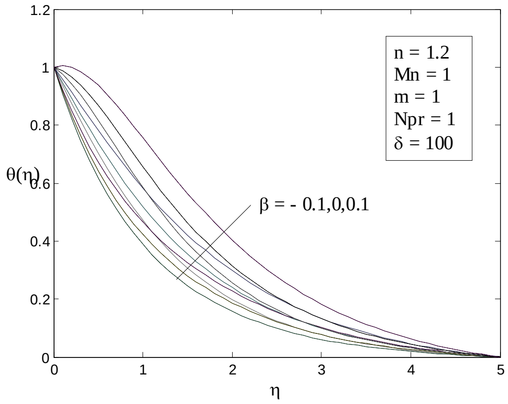
$r = 0, r = 1, r = -1$



7(a)



7(b)



7(c)

Fig.7.Temperature profiles for different values of β and wall temperature parameter (r) with $Mn = 1$

V. Conclusions:

- The effect of increasing values of velocity exponent parameter m is to reduce the horizontal velocity.
- The increasing values of magnetic parameter Mn results in flattering the horizontal velocity profiles and increase the temperature profiles.
- The effectsofPermeability parameter δ is to increase the velocity profiles and decrease the temperature profiles.
- The effect of modified Prandtl number Npr is to decrease the wall temperature gradient.
- The internal heat source/sink parameter β increases the temperature profiles.

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